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Published in:
Proceedings of the 30th Nordic Seminar on Computational Mechanics (NSCM-30)

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

30th Nordic Seminar on Computational Mechanics
NSCM-30
J. Høgsberg. N.L. Pedersen (Eds.)
2017

QUASI-STATIC CONDENSATION OF AEROELASTIC SUSPENSION BRIDGE MODEL

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Key words: Structural Dynamics, Quasi-static Condensation, Aerodynamic Stability, Long Span Bridges.

1 INTRODUCTION

For long span bridges the wind-induced dynamic response is a design driving factor and therefore continuously a subject for detailed analysis. Traditionally both buffeting and stability calculations have been considered in the frequency domain. However, this yields a limitation in accounting for turbulence when considering the stability limit and further it is not possible to account for non-linear effects. These limitations suggest to do simulations of the aeroelastic response of long span bridges in the time domain. For this it is of interest to have an efficient model while still maintaining sufficient accuracy.

This contribution is on quasi-static reduction of an aeroelastic finite element model of a 3000m suspension bridge proposed for crossing Sulafjorden in Norway[1]. The model is intended for stability limit calculation where the representation of higher modes is of less importance. The present contribution demonstrates the application of quasi-static condensation to long suspension bridges as well as introduces an extension of the method to include the full aeroelastic system. This includes considerations on reduction of external wind loading as well as motion-induced forces.

2 AEROELASTIC BRIDGE MODEL

The suspension bridge is depicted in Figure 1(a). The bridge is implemented as a finite element model in Matlab using 3D beam elements for the towers and Green strain truss elements for the cables. The deck to hanger connections are modelled with rigid links and the bridge deck elements are aeroelastic beam elements including aerodynamic properties through additional degrees of freedom in the system.

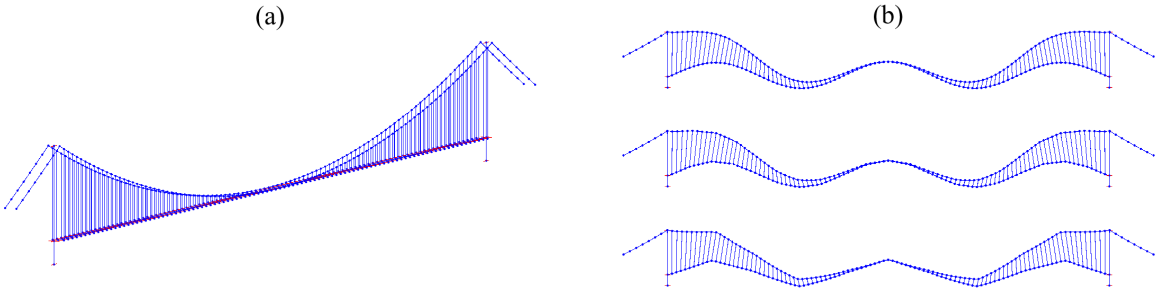

Figure 1: (a) Bridge model (b) Mode shapes: Full model, 1:5 and 1:10 reduction.

The equation of motion for the system including motion-induced forces can be written as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}_m(t) + \mathbf{f}_{ext}(t) \tag{1}$$

where $\mathbf{q}$ is the displacement vector. The coefficient matrices $\mathbf{M} = \mathbf{M}_s + \mathbf{M}_a$, $\mathbf{C} = \mathbf{C}_s + \mathbf{C}_a$ and $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_a$ are the mass, damping and stiffness matrices. Here index s and a refers to structural and aerodynamic property matrices respectively. On the right hand side there is a contribution from the external forces $\mathbf{f}_{ext}$ and from the memory part of the motion-induced forces $\mathbf{f}_m$. To enable the memory part of the motion-induced forces to appear as additional degrees of freedom in the system, a number of j first order differential equations on the form

$$\dot{\mathbf{f}}_{m,j}(t) + \mathbf{D}_j\mathbf{f}_{m,j}(t) = \mathbf{A}_j\mathbf{q}(t) \tag{2}$$

are introduced[2,3]. Here the matrix $\mathbf{A}_j$ and the diagonal matrix $\mathbf{D}_j$ are shape specific property matrices. The aerodynamic properties are implemented as a single term approximation to the Theodorsen flat plate theory.

3 QUASI-STATIC CONDENSATION

The method of quasi-static condensation builds on the master-slave constraint principle, but instead of introducing rigid links between master and slave nodes a stiffness relation is used to describe the master-slave relation:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_s \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{S} \end{bmatrix} \mathbf{q}_d\,, \qquad \mathbf{S} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sd} \tag{3}$$

Here $\mathbf{q}_d$ and $\mathbf{q}_s$ are the displacement vectors of the master and the slave degrees of freedom respectively. The first and second index on the stiffness matrix $\mathbf{K}$ refer to master or slave rows and columns respectively. Extending the method to include motion-induced forces suggest to treat the state proportional aeroelastic terms as the structural mass, damping and stiffness while the memory part of the motion-induced forces are reduced by considering the rate of work. Hereby the reduced form of the equation of motion including motion-induced forces is

$$\bar{\mathbf{M}}\ddot{\mathbf{q}}_d(t) + \bar{\mathbf{C}}\dot{\mathbf{q}}_d(t) + \bar{\mathbf{K}}\mathbf{q}_d(t) = \bar{\mathbf{f}}_m(t) + \bar{\mathbf{f}}_{ext}(t) \tag{4}$$

and the differential equation describing the relation between the system displacements and the memory part of the motion-induced forces is found as:

$$\dot{\bar{\mathbf{f}}}_{m,j}(t) + \gamma_j \bar{\mathbf{f}}_{m,j}(t) = \bar{\mathbf{A}}_j \mathbf{q}_d(t) \tag{5}$$

The differential equation has maintained the form beneficial for implementation assuming that $\mathbf{D}_j = \gamma_j \mathbf{I}$. In equation (4) and (5) the reduced coefficient matrices, external forces and memory forces are found on the general form:

$$\bar{\mathbf{X}} = \mathbf{X}_{dd} + \mathbf{S}^T\mathbf{X}_{sd} + \mathbf{X}_{ds}\mathbf{S} + \mathbf{S}^T\mathbf{X}_{ss}\mathbf{S}\,, \quad \bar{\mathbf{x}} = \mathbf{x}_d + \mathbf{S}^T\mathbf{x}_s \tag{6}$$

where $\mathbf{X} \in \{\mathbf{M}, \mathbf{C}, \mathbf{K}, \mathbf{A}_j\}$ and $\mathbf{x} \in \{\mathbf{f}_{ext}, \mathbf{f}_m, \dot{\mathbf{f}}_m\}$.

4 ACCURACY OF REDUCED SYSTEM

The quasi-static system condensation has been applied to the bridge model and reduced model results are in this section compared to results obtained for the full system. Figure 1(b) shows mode shapes of the 14th still-air mode obtained with the full model, a model reduced to one fifth and to one tenth of the full model size respectively. It is seen that the mode shape obtained with the 1:5 model reduction is very similar to the mode shape obtained with the full model while the 1:10 reduction is resulting in a less smooth mode shape. This implies that the 1:10 reduction is too coarse and is no longer capturing the behaviour of the full model.

Figure 2 shows the natural frequencies for the full model (∘), the 1:5 reduction (∗) and the 1:10 reduction (◇). The left plot shows the natural frequencies for the still-air system and the right plot shows the natural frequencies for the aeroelastic system with wind speed $U = U_{cr}$. Both plots indicate that the 1:5 reduction gives a good representation of modes up to around mode number 15 while the 1:10 reduction shows divergence at mode 9 and up.

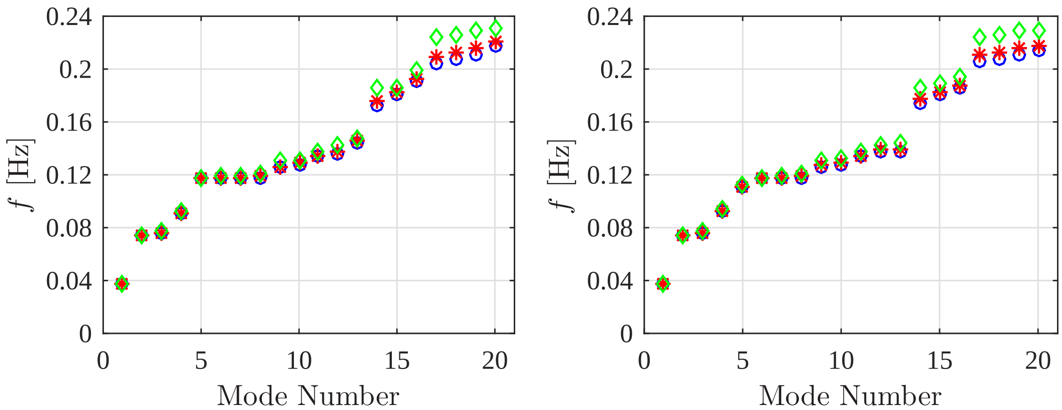

Figure 2: Modal frequencies: (left) still air (right) critical wind speed.

The time domain responses of the full bridge model and the 1:5 reduction are now considered when excited by turbulent wind loading. The mean wind loading influencing the aeroelastic terms is set to $U = 0.6U_{cr}$, the turbulence intensity is $I_u = 0.134$ and the integral length scale is $\lambda = 200$m. Figure 3 shows the drag q_y, the heave q_z and the torsional r_x response for a five minute time interval. It is seen that the response obtained by the reduced model is corresponding well with the response found by the full model. The calculation time for the five minute time history is significantly reduced from 59.3s to 0.7s making use of the reduced model instead of the full model.

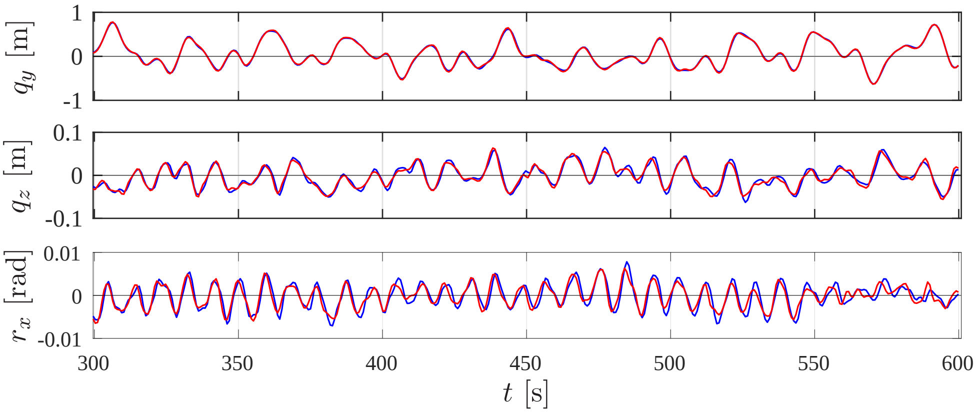

Figure 3: Time response at quarter span: Full model (—), 1:5 reduction (—).

5 CONCLUSIONS

It has been demonstrated that flutter analysis of a typical long suspension bridge can be performed using a model reduced by quasi-static condensation. A reduced model with a number of around 20 master nodes along the bridge deck and corresponding master nodes in the suspension cables has been shown to represent the behaviour of the bridge with sufficient accuracy. A further reduction of the model did not capture the essentials of relevant modes due to the coarseness.

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